Duration Dependence and Unemployment Rate Persistence

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Abstract

The unemployment rate is persistent over the business cycle. However, standard search models contain little internal propagation and predict that, after shocks, the unemployment rate quickly converges to its steady state level. I show that duration dependence in unemployment (the fact that unemployed workers with longer unemployment spells are less likely to find jobs) helps explain the persistence of the unemployment rate. I embed duration dependence in an otherwise standard search model and show that it significantly increases the persistence of the unemployment rate over the business cycle, reconciling the model to the data. Intuitively, after recessions, the composition of the unemployment pool shifts to the long-term unemployed. Because of duration dependence, the long-term unemployed have lower job finding rates, and the shift in composition decreases the aggregate job finding rate, slowing recovery. The magnitude of the effect depends on the extent to which duration dependence is causal rather than a consequence of worker heterogeneity.

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1 Introduction

In the data, the unemployment rate is persistent over the business cycle, particularly after recessions as unemployment slowly declines to its previous level. Its persistence presents a challenge to standard Diamond-Mortensen-Pissarides (henceforth DMP) search models, which struggle to generate realistic unemployment persistence due to little internal propagation. Rather, DMP models predict that, after shocks, the unemployment rate quickly snaps back to its steady state level. In this paper, I use a calibrated search model to show that negative duration dependence can help explain the persistence of the unemployment rate and, when accounted for in the model, helps to reconcile the model with the data. Like others (Gorry et al., 2020; Pries, 2004), I show that a notion of worker heterogeneity can reconcile the facts; in my case, I use heterogeneity in job finding rates that arise from different lengths of time unemployed.

My model is a standard DMP model except that there are two states of unemployment, the high state and the low state, where unemployed workers in the low state are less likely to find a job. Unemployed workers arrive to the low state in two ways. First, if unemployed workers in the high state do not find a job, they move to the low state with a fixed probability. Second, when separated from a job, a fraction of newly unemployed workers begin their unemployment spell in the low state.

Therefore, the model accounts for two reasons why workers with longer unemployment spells are less likely to find a job. First, if there is "pure" duration dependence, then a long unemployment spells causally decreases the job finding rate. Since unemployed workers flow to the low state over their unemployment spell in the model, workers with longer unemployment spells are more likely to be in the low state and less likely to find a job. However, if workers are heterogeneous in job finding rates regardless of unemployment length, workers who are more likely to find a job will find their jobs early on, and workers with longer unemployment spells will be those who had worse job finding rates at the start.

In calibration, I target standard moments as well as the shape of the job finding rate over unemployment duration curve. Quantitatively, the model is flexible enough that the decrease in the job finding rate over unemployment duration can be wholly explained by pure duration dependence, heterogeneity, or a combination of both. In the mixed model, there is a realistic mix of pure duration dependence and heterogeneity. However, I can make simple adjustments to the model, and recalibrate models with only pure duration dependence and only heterogeneity.

The main result is that duration dependence increases the persistence of the unemployment rate over the business cycle. The mechanics behind the result are straightforward. During a recession, job finding rates decrease, so the composition of the unemployment pool shifts towards the long-term unemployed. Because of duration dependence, the long-term unemployed are less likely to find a job. Thus, duration dependence drags down the job finding rate of the unemployment pool as a whole, and the unemployment rate recovers slowly. Quantitatively, in the mixed model, the unemployment rate is significantly more persistent, but not quite to the level of persistence in the data.

Using my alternative models, I show that the effect is significantly stronger with more pure duration dependence. In other words, I find weaker aggregate effects if the observed decline in job finding rates over unemployment duration is entirely due to inherent characteristics of workers. Thus, this paper points out that distinguishing between "pure" duration dependence and unobserved heterogeneity is of some importance in macroeconomics.

One caveat is that duration dependence slows recovery after productivity shocks but not after job separation shocks. If job separations suddenly increase, the inflows into unemployment *improve* the composition of the unemployment pool since, by definition, newly unemployed workers are short-term unemployed and more likely to find a job. Thus, firms post more jobs, which mitigates the effect of the shock. To the extent to which the COVID-19 recession can be characterized by a job separation shock (Cajner et al., 2020), the theory may help to explain the relatively fast recovery of the unemployment rate for 2020-2021.

Policymakers have recently expressed concern about the aggregate effects of duration dependence. A 2014 White House report titled "Addressing the Negative Cycle of Long-Term Unemployment" claims that "the cycle of long-term unemployment hampers the economy at large, depressing aggregate demand and resulting in the underutilization of productive resources" (White House, 2014). After the Great Recession, the Congressional Budget Office stated that duration dependence "currently accounts for about a quarter of a percentage point of the increase in unemployment during and following the recession" (Congressional Budget Office, 2012).

The remainder of this paper is as follows. I contextualize the paper within the literature in Section 2. Section 3 establishes the empirical facts which motivate my analysis. I describe and calibrate the model in Sections 4 and 5. Section 6 presents my results. I conclude in Section 7.

2 Related Literature

As is pointed out in Pries (2004) and Shimer (2005), DMP models following Mortensen and Pissarides (1994) fail to generate realistic persistence of the unemployment rate.[1] My solution is similar to Gorry et al. (2020), which shows that changes in the skill composition in the labor force over the business cycle will amplify shocks and increase persistence. Instead of heterogeneity in skill, my theory only requires that workers lose job finding probability over the course of their unemployment spell. Pries (2004) focuses on the high risk of job loss for new workers; my focus in on the inverse, or the low job finding rate for the long-term unemployed. However, both elements generate more persistence in the unemployment rate.

I also contribute to a strand of literature that relates duration dependence with aggregate labor market dynamics. One message from this strand is that a notion of heterogeneity or duration dependence is crucial for understanding long-term unemployment. For

[1] The same is true in Merz (1995), which combines DMP labor market search with a real business cycle model.

example, Kroft et al. (2016) shows that duration dependence decreased the aggregate job finding rate during the Great Recession, slowing recession.[2] My result is closest to Pissarides (1992), which shows that the loss of skills during unemployment generates more persistence in unemployment.

Generally, the driving force behind my results is that the composition of the unemployment pool shifts during recessions.[3] The same force is explored in Ferraro (2018), Ravenna and Walsh (2012), and Wiczer (2015), though duration dependence is not a key mechanism. Other related papers include Ahn and Hamilton (2020) and Hornstein (2012), which account for duration dependence in analyzing outflows from unemployment to employment.[4]

Finally, my results speak to another well-known problem that DMP models struggle to match business cycle volatility in matching efficiency unless the matching efficiency multiplier (typically denoted by μ) fluctuates wildly (Barnichon and Figura, 2015; Lubik, 2009). In my model, unemployment composition is an endogenous channel through which matching efficiency decreases during recessions.

3 Empirical Facts

My analysis is driven by two empirical facts, one macro and one micro. The macro fact is that the unemployment rate is persistent. The micro fact is that unemployed workers are less likely to find jobs later in their unemployment spell. In this paper, I show that the micro fact helps explain the macro fact.

First, unemployment is persistent. As illustrated in Figure 1, unemployment spikes during recessions, but slowly decreases between recessions. Quantitatively, the autocor-

[2]*During* the Great Recession, Elsby et al. (2010) and Aaronson et al. (2010) conjectured that duration dependence would slow the recovery of the unemployment rate.

[3]Though I often mention recessions, my analysis is about unemployment persistence over the entire business cycle, which could in principle include positive shocks. However, my illustrative focus is on negative shocks. In a model where shocks are only recessionary (Dupraz et al., 2019), an analysis of unemployment persistence is simultaneously an analysis of slow recoveries.

[4]Also see Jarosch and Pilossoph (2019), which provides a counter-argument that duration dependence is not relevant in the aggregate.

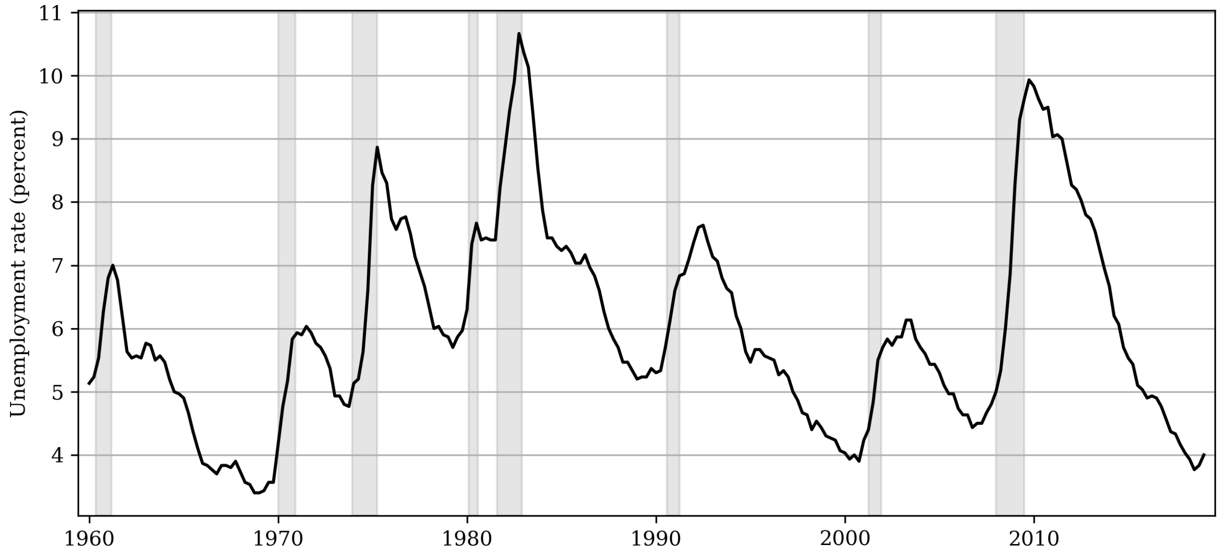

Figure 1: Unemployment rate over time

Source: US Bureau of Labor Statistics (BLS). Unemployment rate is quarterly. Gray bars denote NBER recessions.

relation of the unemployment rate is 0.975 (see Table 4).[5]

Second, Figure 2 illustrates that the probability of finding a job decreases over the unemployment spell, especially early on. On average, a worker who has been unemployed for less than a month has a 50% chance of finding a job, while a worker who has been unemployed for over eight months has a 20% chance of finding a job.

In the unemployment context, "pure" negative duration dependence refers to the causal notion that unemployment duration negatively affects an individual's probability of finding a job.[6] Using a resume audit study, Kroft et al. (2013) find that a job applicant with under one month of unemployment is 45% more likely to receive a callback for a job interview than an applicant who has been unemployed for eight months with an otherwise identical resume.[7]

[5]The unemployment rate is also asymmetric. For theories behind the asymmetry of the unemployment rate, see Ferraro (2023, 2018), Dupraz et al. (2019), and Rudanko (2024).

[6]As is common, I use the terms "negative duration dependence" and "duration dependence" interchangeably, though "negative duration dependence" is more precise.

[7]For more experimental evidence, see Eriksson and Rooth (2014), Farber et al. (2019), and Oberholzer-Gee (2008).

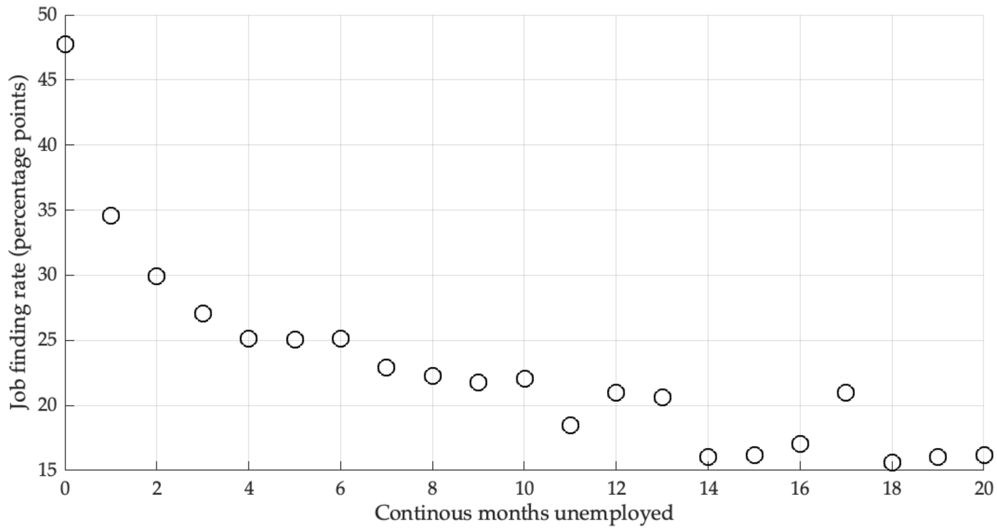

Figure 2: Job finding rate by unemployment duration

Source: Author's calculations using CPS micro data for 1978-2019. The y-intercept is the probability that a worker finds a job before being unemployed for a full month.

The sources of negative duration dependence are unclear. One hypothesis is that workers lose skills while unemployed, rendering them less productive upon returning to the workforce (Ljungqvist and Sargent, 1998; Edin and Gustavsson, 2008). Another theory is that firms may interpret long duration as a signal of inefficiency and statistically discriminate against such workers (Blanchard and Diamond, 1994; Lockwood, 1991). And long unemployment duration may be associated with worker discouragement and lower search intensity (Faberman and Kudlyak, 2019; Krueger and Mueller, 2011).

In my model, I take productivity to be constant across workers, so explanations of pure duration dependence which rely upon productivity are incompatible with my framework. One theory that is compatible with my model relates to recall hiring. Over 40% of unemployed workers who separate into unemployment return to previous employer, and the probability of being recalled declines sharply over the unemployment spell. In fact, the rate of exit from unemployment to a different employer is only slightly decreasing over the unemployment spell (Fujita and Moscarini, 2017). Another theory is that long-term unemployed workers have exhausted the job opportunities in their social net-

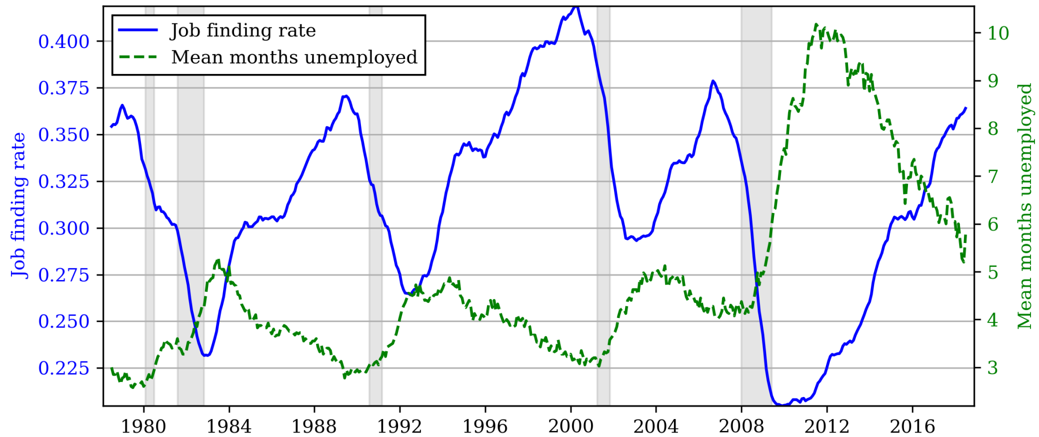

Figure 3: Aggregate job finding rate and mean unemployment duration

Mean unemployment duration source: BLS. Job finding rate source: Author's calculations using CPS data. Both series are quarterly. Gray bars denote NBER recessions. The job finding rate is monthly.

work.[8]

However, Figure 2 does not necessarily imply the existence of pure, or causal, negative duration dependence. The downward-sloping curve could merely be a result of worker heterogeneity.[9] If job searchers are heterogeneous in job finding rates regardless of unemployment length, workers who are more likely to find a job are also more likely to find a job earlier in their unemployment spell. Thus, workers with a higher unemployment duration are more likely to be those with a worse job finding probability at the beginning. My model accounts for both pure duration dependence and heterogeneity, and I experiment with assigning different levels of blame to the two factors.[10]

Figure 3 illustrates that the micro fact holds in the aggregate; an increase in mean un-

[8]In a slightly different story, Calvó-Armengol and Jackson (2004) show social networks contribute to duration dependence because those within the same social network are more likely to be unemployed at the same time.

[9]Such heterogeneity is sometimes referred to as "unobserved heterogeneity." I avoid the "unobserved" term in this paper because differences in job finding are the only source of heterogeneity in my model.

[10]Distinguishing pure state dependence from unobserved heterogeneity is an old puzzle in econometrics (Heckman, 1991). Some studies suggest that the decrease in job finding rates over unemployment duration is mostly explained by unobserved heterogeneity (Alvarez et al., 2023; Abbring et al., 2002; Machin and Manning, 1999). On the other hand, this view is difficult to reconcile with studies which find that observable differences between workers are not important predictors of unemployment duration (Elsby et al., 2010; Krueger et al., 2014).

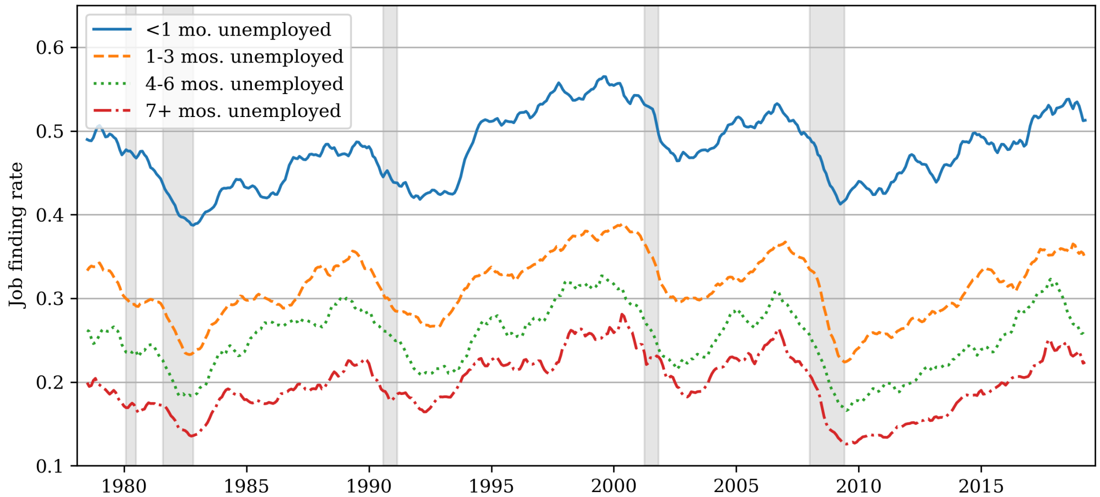

Figure 4: Job finding rate by unemployment duration over time

Source: Author's calculations using CPS data. Gray bars denote NBER recessions. Job finding rates are monthly. All series are smoothed to quarterly.

employment duration is associated with a decrease in the aggregate job finding rate. Also note that, like the unemployment rate as a whole, the job finding rate is slow to recover after recessions. Thus, Figure 3 summarizes the mechanism in my model. As the average unemployment duration increases, duration dependence puts downward pressure on the aggregate job finding rate, making the job finding rate slower to recover. The slow recovery of the job finding rate then drives the slow recovery of unemployment.[11]

The gap in job finding probability by unemployment duration is relatively constant over the business cycle. As Figure 4 shows, job finding rates for different unemployment durations move roughly in parallel. In addition to motivating a key assumption of the model, Figure 4 suggests that the aggregate job finding rate can fluctuate as a result of changes in the composition of the unemployment pool by unemployment duration.

[11]Note that I am only referring to unemployment outflows, not inflows. However, there is empirical support for the idea that recovery is driven more by job finding (outflows) than job separations (inflows). For the view that all unemployment fluctuations are mostly driven by job finding, see Shimer (2012), Hall (2005), and Elsby et al. (2009). For the view that separations are important for fluctuations in unemployment but that recovery is still mostly driven by outflows, see Elsby et al. (2013), Barnichon (2012), Fujita (2011), Elsby et al. (2010), and Fujita and Ramey (2009).

4 Model

I analyze duration dependence using an otherwise-standard DMP search model with unemployment heterogeneity. There are two states of unemployment, the high state and the low state, where the only distinguishing feature is that those in the low state are less likely to find a job than those in the high state. In each period of unemployment, there is a probability that a unemployed worker in the high state transitions to the low state. Thus, duration dependence enters the model because workers who have been unemployed longer are more likely to be in the low state and therefore less likely to find a job. Following negative shocks, the composition of the unemployment pool shifts toward the low state, decreasing the aggregate job finding rate, and slowing recovery.

4.1 Environment

The model environment closely resembles the standard DMP search model in discrete time (Pissarides, 2000). There is a measure one of workers and a continuum of firms, both of which discount the future by discount factor β. Workers and firms are risk neutral and infinitely lived. The sole market is for labor. Firms post v_t vacancies in time t to maximize expected future profit and use hired labor to produce a single output good, the price of which is normalized to one. Each employed worker produces A_t output in exchange for a wage w_t. Workers are either employed or unemployed, and open job vacancies and unemployed workers match randomly. Jobs are destroyed exogenously with probability λ_t, whereupon workers become unemployed.

My model differs from the standard search model by allowing for heterogeneous unemployment and duration dependence. Unemployed workers are either in the high state or the low state. The total unemployment rate is $u_t = u_t^L + u_t^H$, the sum of unemployed workers in the high state, u_t^H, and the low state, u_t^L. The employment rate is $n_t = 1 - u_t$.

Some workers begin their unemployment spell in the low state; others flow from the high state to the low state due to pure duration dependence. With probability ζ, newly separated workers begin their unemployment spell in the low state. With probability ϕ, high-state workers who are unable to find a job flow to the low state. Equivalently, one can say that a ζ fraction of workers will always begin unemployment in the low state, and the rest of the worker pool can enter the low state of unemployment as a result of pure duration dependence, but will begin every unemployment spell in the high state. Thus, ζ generates heterogeneity in job finding rates and ϕ generates duration dependence.

The probability of finding a job in the low state, f_t^L, is a constant fraction $\gamma \in (0,1)$ of the probability of finding a job in the high state f_t^H, $f_t^L = \gamma f_t^H$. I refer to γ as the low state penalty.

My framework nests the standard DMP model when $\zeta = \phi = 0$, a benchmark I refer to as the DMP model. If $\zeta = \phi = 0$, then all unemployed workers are in the high state.

I simulate the model response to exogenous shocks to productivity, A_t, and the separation rate, λ_t. Both are given by AR(1) processes in deviation from the steady state,

$$s_{t+1} - s = \rho_s \left(s_t - s\right) + \varepsilon_s, \quad \varepsilon_s \sim N(0, \sigma_s^2)$$

for $s \in \{A, \lambda\}$ where ρ_s is persistence and σ_s is volatility. In practice, I will feed the model shocks to one while holding the other constant.

4.2 Laws of Motion

The inflows and outflows of unemployment are summarized by two laws of motion. Workers flow out of the high state either by finding a job with probability f_t^H or, if they do not find a job, by moving to the low state with probability ϕ. The total number of workers flowing into unemployment is $\lambda_t n_t$, a fraction $1 - \zeta$ of which begin unemployment in the high state. Combining inflows and outflows, the law of motion for high-state unemployment is

$$u_{t+1}^H = \left(1 - f_t^H\right) u_t^H - \phi \left(1 - f_t^H\right) u_t^H + (1 - \zeta)\, \lambda_t n_t. \tag{1}$$

Workers in the low state of unemployment cannot move to the high state, so workers in the low state only flow out by finding a job with probability f_t^L. The low state receives inflows from the fraction ζ of workers who lose their job as well as those who flow from the high state to the low state. The law of motion for low-state unemployment is

$$u_{t+1}^L = u_t^L \left(1 - f_t^L\right) + \phi \left(1 - f_t^H\right) u_t^H + \zeta \lambda_t n_t. \tag{2}$$

4.3 Matching Functions

I use a matching technology which generates a constant gap in job finding probabilities between the high state and low state, $f_t^L = \gamma f_t^H$, and nests the standard Cobb-Douglas matching function.[12] The following scheme with two Cobb-Douglas-esque matching functions, one for each unemployment state, is intuitive and satisfies both requirements.

At time t, the number of matches formed between open job vacancies and high-state unemployed workers is

$$m^H \left(u_t^H, u_t^L, v_t\right) = \mu \frac{u_t^H}{u_t} \left(u_t^H + \gamma u_t^L\right)^\alpha v_t^{1-\alpha} \tag{3}$$

where μ is matching efficiency and α is an elasticity parameter. For intuition, contrast Equation (3) with the standard Cobb-Douglas matching function, $\tilde{m}\left(u_t, v_t\right) = \mu u_t^\alpha v_t^{1-\alpha}$. My function includes two new terms. The first, u_t^H / u_t, represents the portion of the unemployment pool to which this matching function applies (a necessary element since vacancies are posted for both worker types). The second, $u_t^H + \gamma u_t^L$, replaces u_t and represents the weighted "matchability" of the unemployment pool.

The number of matches between open job vacancies and unemployed workers in the low state is

$$m^L \left(u_t^H, u_t^L, v_t\right) = \gamma \mu \frac{u_t^L}{u_t} \left(u_t^H + \gamma u_t^L\right)^\alpha v_t^{1-\alpha}, \tag{4}$$

the same expression as (3) but scaled down by γ and applied to the u_t^L / u_t part of the

[12]The assumption of a constant gap in finding rates is motivated by the data in Figure 4.

unemployment pool.

Let $\theta_t \equiv v_t/u_t$ denote labor market tightness and $x_t \equiv u_t^H/u_t$ denote the fraction of the unemployment pool in the high state. The job finding rate for unemployed workers in the high state is

$$f_t^H \equiv \frac{m^H\left(u_t^H, u_t^L, v_t\right)}{u_t^H} = \mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha}\theta_t^{1-\alpha}. \tag{5}$$

Similarly, the job finding rate for unemployed workers in the low state is

$$f_t^L \equiv \frac{m^L\left(u_t^H, u_t^L, v_t\right)}{u_t^L} = \gamma\mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha}\theta_t^{1-\alpha}. \tag{6}$$

These matching functions successfully generate $f_t^L = \gamma f_t^H$.

Unemployment composition ,x_t, is the key element that generates persistence beyond the DMP model. Define the aggregate (or average) job finding rate as

$$f_t \equiv x_t f_t^H + \left(1-x_t\right) f_t^L = \mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha+1}\theta_t^{1-\alpha}. \tag{7}$$

Note that f_t is increasing in x_t as well as θ_t.[13] If x_t is persistent over the business cycle, then f_t will be more persistent. So, if x_t decreases during recessions as unemployment spells become longer, x_t will decrease the aggregate job finding rate and slow recovery.

From the firm's perspective, the probability that an open position is filled by an unemployed worker from the high state is $h_t^H \equiv m^H\left(u_t^H, u_t^L, v_t\right)/v_t$, and the probability that an open position is filled by a worker from the low state is $h_t^L \equiv m^L\left(u_t^H, u_t^L, v_t\right)/v_t$. The aggregate hiring rate is

$$h_t \equiv h_t^L + h_t^H = \mu\left(x_t(1-\gamma)+\gamma\right)^{\alpha+1}\theta_t^{-\alpha}. \tag{8}$$

Like the aggregate job finding rate in Equation (7), the aggregate hiring rate is increasing

[13]In the DMP model, $\tilde{f}_t = \mu\theta_t^{1-\alpha}$.

in x_t.[14]

4.4 Wages

Wages are determined by a Nash bargaining surplus sharing rule. As such, I first present value functions. For a worker, the value of unemployment in the high state is

$$U_t^H = z + \beta \left[f_t^H E_{t+1} + \left(1 - f_t^H\right) \left(\phi U_{t+1}^L + (1-\phi) U_{t+1}^H \right) \right] \tag{9}$$

where E_t is the worker's value of employment and z is the flow utility of unemployment. With probability f_t^H, the worker finds a job and begins work in the next period; otherwise, the worker remains unemployed. Conditional on not finding a job, the worker begins the next period in the low state of unemployment with probability ϕ; otherwise, the worker remains in the high state.

The value of unemployment in the low state is

$$U_t^L = z + \beta \left[f_t^L E_{t+1} + \left(1 - f_t^L\right) U_{t+1}^L \right]. \tag{10}$$

An unemployed worker in the low state cannot move to the high state and is less likely to find a job because $f_t^L < f_t^H$.

Next, for the worker, the value of employment is

$$E_t = w_t + \beta \left[(1-\lambda_t) E_{t+1} + \lambda_t \left(\zeta U_{t+1}^L + (1-\zeta) U_{t+1}^H \right) \right]. \tag{11}$$

The worker earns the wage w_t. With probability λ_t, the worker is separated from their job; otherwise, the worker remains employed. Upon separation, the worker begins the unemployment spell in the low state with probability ζ; otherwise, the worker begins unemployment in the high state.

[14] In the DMP model, $\tilde{h}_t = \mu \theta_t^{-\alpha}$.

From the firm's perspective, the value of a filled job is

$$J_t = A - w_t + \beta \left[(1 - \lambda_t) J_{t+1} + \lambda_t V_{t+1} \right].$$

where V_t is the value of an open job vacancy. In the current period, the firm earns the value of the worker's production minus the wage. With probability $1 - \lambda_t$, the job stays intact for another period; with probability λ_t, the match is destroyed, and the job becomes an open vacancy. The value of an open job vacancy is

$$V_t = -\kappa + \beta \left[h_t J_{t+1} + (1 - h_t) V_{t+1} \right]$$

where κ is the cost of posting a vacancy each period.

I assume free entry of firms in the labor market. In equilibrium, profit maximization requires that the total discounted value of a vacancy equals zero, $V_t = 0$ for all t. So, in equilibrium, the previous two equations become

$$J_t = A - w_t + \beta (1 - \lambda_t) J_{t+1} \tag{12}$$

and

$$\kappa = \beta h_t J_{t+1}. \tag{13}$$

Equation (13) (the free entry condition) determines the number of vacancies posted in equilibrium. Combining Equations (13) and (8), it is clear that unemployment composition will affect job posting. All else equal, if the unemployment pool has many workers in the low state (x_t is low), fewer jobs will be posted (v_t and θ_t will be low).

Wages are determined by Nash bargaining where the worker has bargaining weight ψ. Therefore, the worker and firm split the total surplus – worker surplus plus firm surplus – generated from the match. Firm surplus is the value of a filled job minus the value of a vacancy, $J_t - V_t = J_t$; worker surplus is the value of employment, E_t, minus the value of unemployment for that worker.

However, a worker's value of unemployment (i.e., their threat level) depends upon their status in the current period. A worker exists in one of three categories: currently employed, high-state unemployed, or low-state unemployed. These categories determine a worker's value of unemployment and thus their threat level. Since low-state unemployed workers have a lower value of unemployment than high-state unemployed workers, low-state workers are more desperate and willing to work for a lower wage.[15]

Using the value functions above, if a worker is currently employed, their value of unemployment is the value of losing their job, $\zeta U_t^L + (1-\zeta) U_t^H$. If a worker is high-state unemployed, their value of remaining unemployed is $\phi U_t^L + (1-\phi) U_t^H$. And if a worker is low-state unemployed, their value of remaining unemployed is U_t^L.

I assume that all workers are paid the same wage.[16] For all workers to earn the same wage, I must assume that firms cannot discern between workers. (Either worker status is unobservable for firms or firms are constrained to pay the same wage to all workers.) So, job vacancies are posted for the unemployment pool as a whole and firms are randomly matched with workers.

More precisely, I assume that the firm knows the composition of the workforce but does not know the type of worker with which it is negotiating. Then, I use the Nash bargaining solution with incomplete information derived in Harsanyi and Selten (1972). Thus, the firm does know the status of the worker with which it is negotiating, but knows that the probability that a worker is currently working, unemployed in the high state, or unemployed in the low state is n_t, u_t^H, and u_t^L, respectively. Writing E_t and J_t as functions

[15]There is empirical evidence which suggests that workers with longer unemployment spells have lower reservation wages (Krueger and Mueller, 2016) and earn lower wages once they find a job (Schmieder et al., 2016).

[16]My results are robust to this assumption. I find that wage bargaining schemes where firms can pay different types of workers different wages generate very similar predictions.

of the wage, the wage solves

$$w_t = \arg\max_{\hat{w}_t} \left\{ \left(\left[E_t\left(\hat{w}_t\right) - \left(\zeta U_t^L + (1-\zeta)\, U_t^H \right) \right]^{n_t} \right. \right.$$
$$\left. \left. \times \left[E_t\left(\hat{w}_t\right) - \left(\phi U_t^L + (1-\phi)\, U_t^H \right) \right]^{u_t^H} \left[E_t\left(\hat{w}_t\right) - U_t^L \right]^{u_t^L} \right)^{\psi} J_t\left(\hat{w}_t\right)^{1-\psi} \right\}.$$

Note that the probabilities for worker status (n_t, u_t^H, and u_t^L) appear as exponents in the worker's surplus term. Taking logs and maximizing, the equilibrium wage w_t solves

$$(1-\psi)\,\frac{1}{J_t(w_t)}$$
$$= \psi \left(\frac{n_t}{E_t(w_t) - \left[\zeta U_t^L + (1-\zeta)\, U_t^H\right]} + \frac{u_t^H}{E_t(w_t) - \left[\phi U_t^L + (1-\phi)\, U_t^H\right]} + \frac{u_t^L}{E_t(w_t) - U_t^L} \right). \tag{14}$$

Again, the model collapses to DMP with standard Nash bargaining if $\zeta = \phi = 0$.

5 Calibration

5.1 External Calibration

External parameters choices are listed in in Table 1. The model period is one month. I set β to 0.9967 in accordance with a risk-free real interest rate of 4%. The flow utility of unemployment z is set to 0.73 according to the calculations in Mortensen and Nagypal (2007) and the vacancy creation cost κ is set to 0.3 according to Michaillat (2012). I set α and ψ equal to 0.6.[17] I follow Coles and Moghaddasi Kelishomi (2018) for shock process parameters $\{\rho_A, \sigma_A, \rho_\lambda, \sigma_\lambda\}$. A is normalized to 1.

[17] With $\alpha = 0.6$, I seek to anchor to the macro standard of 0.5 and acknowledge the larger estimates in micro literature such as Petrongolo and Pissarides (2001) and Lange and Papageorgiou (2020). Similarly, ψ is set to 0.6 by combining the macro standard of 0.5 with the larger results in Jäger et al. (2020).

Table 1: External parameters

Parameter	Meaning	Value	Explanation/source
β	Discount factor	0.9967	4% annual risk-free rate
z	Flow utility of unemployment	0.73	Mortensen and Nagypal (2007)
κ	Vacancy creation cost	0.3	Michaillat (2012)
α	Matching function elasticity	0.6	Petrongolo and Pissarides (2001), Lange and Papageorgiou (2020)
ψ	Worker bargaining weight	0.6	Jäger et al. (2020)
A	Steady state productivity	1	Normalization
ρ_A	Productivity shock persistence	0.965	Coles and Moghaddasi Kelishomi (2018)
σ_A	Productivity shock standard deviation	0.007	Coles and Moghaddasi Kelishomi (2018)
ρ_λ	Separation shock persistence	0.875	Coles and Moghaddasi Kelishomi (2018)
σ_λ	Separation shock standard deviation	0.042	Coles and Moghaddasi Kelishomi (2018)

External parameter choices.

5.2 Internal Calibration

I internally calibrate λ, γ, ϕ, ζ, and μ by matching the steady state unemployment rate u, job finding rate f, and job finding rate as a function unemployment duration $f(\tau)$ in the data. I describe the data used in Appendix A and explicitly list calibration targets in Appendix B.3.

First, I first calibrate λ according to the steady state relationship

$$u = \frac{\lambda}{\lambda + f} \tag{15}$$

which results in $\lambda = 0.02$. I estimate the four remaining parameters using simulated method of moments.[18]

The novelty in my calibration strategy is that I target the job finding rate as a function of unemployment duration, $f(\tau)$, as plotted earlier in Figure 2. To do so, I fit a curve to

[18]For the DMP model, I set μ so that $f = \mu\theta^{1-\alpha}$ where θ is the same as the heterogeneous model; $\phi = \zeta = 0$ and γ is redundant.

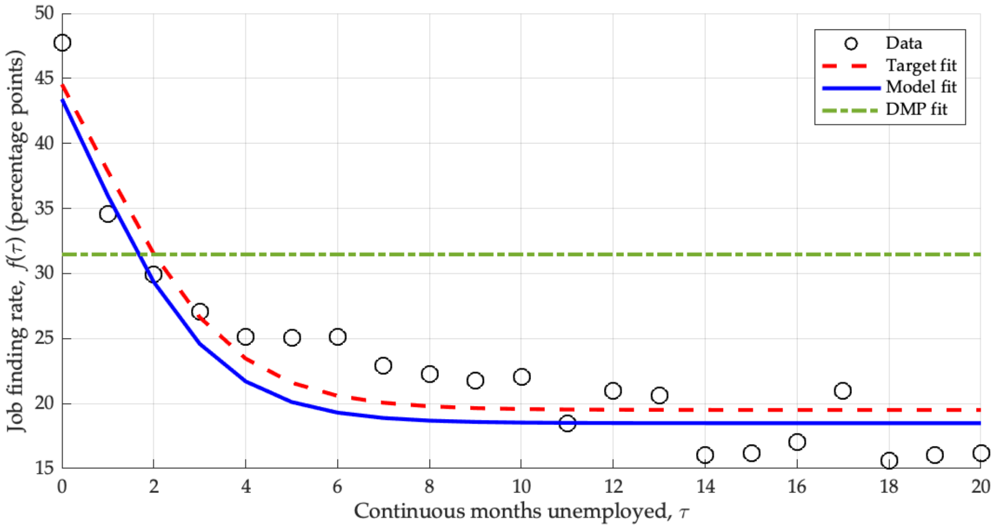

Figure 5: Fit of job finding rate by unemployment duration

Data is identical to Figure 2. Target fit is the curve which fits the data under the functional form of $f(\tau)$ implied by the model of this paper. Model fit is the resulting $f(\tau)$ curve after calibration. The fit looks identical for model variant. DMP fit refers to the fit of $f(\tau)$ in the standard DMP model where $\phi = \zeta = 0$.

the data for $f(\tau)$, then target the three coefficients of the curve. For details, see Appendix B.1. Figure 5 plots the $f(\tau)$ curve and the model fit. Note that the standard DMP model assumes that the job finding rate is constant across all unemployment durations, relegating $f(\tau)$ to a horizontal line, whereas my model matches captures the downward sloping job finding rate.

However, if I only target u, f, and $f(\tau)$, then ϕ and ζ are jointly identified, but not separately identified. Recall that it is unclear whether the downward-sloping $f(\tau)$ curve is caused by pure duration dependence (ϕ) or heterogeneity (ζ). The same is true in my model. In fact, the $f(\tau)$ curve can be generated entirely by negative duration dependence, entirely by heterogeneity, or by infinite combinations of the two. Figure 6 illustrates the situation. The downward-sloping curve is the locus of (ϕ, ζ) pairs that fit the data equally well.

I exploit the ambiguity to separate the aggregate effects of duration dependence from heterogeneity. I consider three versions of the model, two of which are extreme cases. For the first model, which I call the "pure duration dependence model," I assume that pure

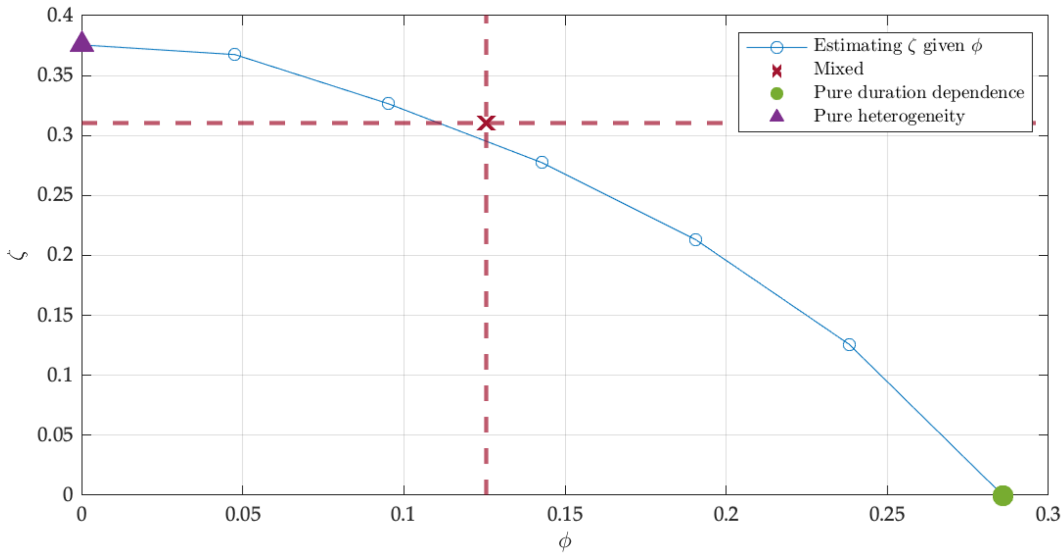

Figure 6: Joint identification of duration dependence and unobserved heterogeneity

Each point on the curve is a different calibration. The line plots calibrated values of ζ when restricting ϕ. ζ represents unobserved heterogeneity and ϕ represents pure duration dependence. The mixed model point is the calibrated (ϕ, ζ) pair when targeting additional moments from van den Berg and van Ours (1996). It represents a realistic relationship between ϕ and ζ. The other two points are extreme cases. If $\zeta = 0$, only pure duration dependence generates the negative relationship between unemployment duration and job finding rate. If $\phi = 0$, only heterogeneity generates the negative relationship.

duration dependence is the only cause of the downward-sloping job finding rate. As such, I set ζ equal to zero, cutting off heterogeneity as a channel of duration dependence, and calibrate the remaining parameters. The pure duration dependence model corresponds to the green circle on the horizontal axis in Figure 6. Then, I assume the opposite; I assume that pure duration dependence is nonexistent and that the downward-sloping $f(\tau)$ curve is due entirely to heterogeneity. To do so, I set ϕ equal to zero and calibrate the remaining parameters. The "pure heterogeneity model" corresponds to the purple triangle in Figure 6.

The third model, which I call the "mixed model", approximates a realistic mix of pure duration dependence and heterogeneity. I target additional moments estimated in van den Berg and van Ours (1996), a paper which quantifies the effects of pure duration dependence and heterogeneity. Appendix B.2 explains how I incorporate the new targets.

Table 2: Calibrated parameters

Parameter	Meaning	Model			
		DMP	Mixed	Pure duration dependence	Pure heterogeneity
γ	Low state penalty		0.34	0.43	0.33
ϕ	Transition rate	0.00	0.13	0.29	0.00
ζ	Initial low state	0.00	0.31	0.00	0.38
μ	Match efficiency	0.40	0.98	0.67	1.04

The mixed model includes moments from van den Berg and van Ours (1996). These moments are not targeted in the other three models; in these models, either ζ, ϕ, or both are restricted to zero.

The mixed model corresponds to the red X in Figure 6; both pure duration dependence and heterogeneity play significant roles.

In the Appendix, I consider one more model which I call the "Hagedorn-Manovkii" model. It is identical to the mixed model but uses the parameter values for the flow utility of unemployment, z, and the worker bargaining weight, ψ, from Hagedorn and Manovskii (2008). These parameter values do not change my persistence results but, as explained in Hagedorn and Manovskii (2008), they significantly increase unemployment volatility.

5.3 Calibration Results

Table 2 lists the resulting parameters for all calibrations. Each calibration fits the data well. See Appendix B.3 for model fit and steady state endogenous variables across calibrations.

First, note that the job finding penalty in the low state is large. In the mixed model, unemployed workers in the low state are 35% as likely to find a job as unemployed workers in the high state.

Second, many unemployed workers are in the low state. Given ζ in the mixed model, workers have a 23% chance of beginning their unemployment spell in the low state, and given ϕ, those that begin in the high state have a 13% chance of moving to the low state each period. Compared with the estimated steady state fraction of unemployed workers

Table 3: Unemployment impulse response function statistics

Statistic	DMP	Mixed	Pure duration dependence	Pure heterogeneity
Peak t	8	13	15	10
Half peak t	31	40	44	37
Half t - peak t	23	27	29	27
Peak	0.00046	0.00046	0.00058	0.00037

Data correspond to unemployment impulse response functions in Figure 7 Panel A. The first two rows show how many time periods transpire after the shock until the unemployment rate reaches its peak or half-peak. IRFs are measured in deviations from the steady state.

in the high state, $x = 0.43$ (see Table 5 in Appendix B.3), we gather that less than a quarter of workers begin unemployment in the low state, but at a point in time, over half of unemployed workers are in the low state. In words, most unemployed workers find jobs quickly, but most of the unemployment pool consists of workers who will probably not find jobs quickly.[19] This paper suggests that the relative size of the latter group has significant business cycle implications.

6 Results

6.1 Duration Dependence and Unemployment Rate Persistence

My main result is that duration dependence, particularly pure duration dependence, increases the persistence of the unemployment rate over the business cycle. In this section, I derive my main result by comparing the persistence of the unemployment rate in response to productivity shocks using different model variants.[20] Figure 7 plots impulse response functions (IRFs) for different models after a negative productivity shock, and Table 3 reports statistics associated with the unemployment IRF in Panel A. The four lines correspond to the four model variants: the mixed calibration with a mix of pure duration

[19] This notion is empirically compatible with Morchio (2020) which finds that 2/3 of prime-age unemployment is accounted for by 10% of workers.

[20] The results using productivity shocks are equivalent to those using discount factor (β) shocks.

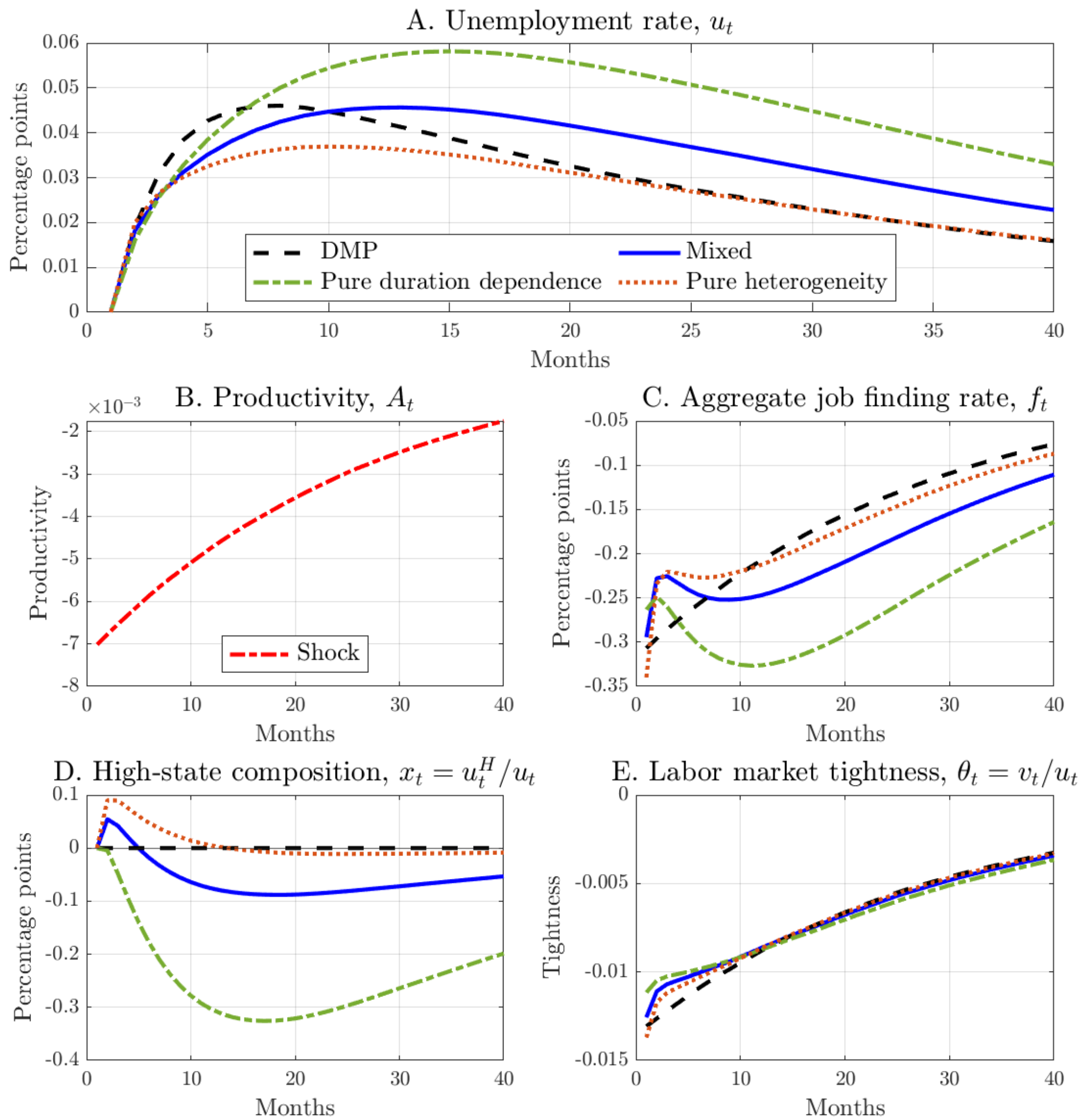

Figure 7: Impulse response functions by calibration after negative productivity shock

All impulse responses are measured in deviations from the steady state. DMP refers to the the model without unemployment heterogeneity. The baseline calibration is a realistic combination of pure duration dependence and heterogeneity; the pure duration dependence calibration attributes all differences in job finding to duration dependence; and the pure heterogeneity calibration attributes all differences in job finding to heterogeneity. Panel B plots the exogenous shock; all other panels plot endogenous responses.

dependence and heterogeneity, the standard DMP model ($\phi = \zeta = 0$), the pure duration dependence model ($\zeta = 0$), and the pure heterogeneity model ($\phi = 0$).[21]

First, contrast the response of the unemployment rate between the mixed model and the DMP model. Whereas in the DMP model, unemployment peaks 8 months after the shock, in the mixed model, it does not peak until 15 months after the shock. Furthermore, the number of months from peak to half-peak is 36 in the mixed model and 30 in the DMP model, implying that unemployment also declines at a slower rate. Both the later peak and the slower recovery rate imply more persistence.

The effect is significantly stronger if pure duration dependence, rather than heterogeneity, drives the negative relationship between job finding rates and unemployment duration. In the pure duration dependence model, the unemployment rate peaks later and the rate of recovery is slower that in the pure heterogeneity model. Also note that the peak is much higher with pure duration dependence, which suggests that pure duration dependence contributes to amplification in addition to propagation. The mixed model is somewhere between the two extremes.

The other panels in Figure 7 illustrate the intuition for why duration dependence increases persistence. Panel B plots he negative productivity shock. λ_t is fixed; thus, the inflow rate to unemployment is fixed, so unemployment fluctuations are driven entirely by the job finding rate, or the rate of outflow from unemployment. Therefore, the size and shape of the aggregate job finding rate in Panel C is responsible for the size and shape of the unemployment rate in Panel A. Aggregate job finding, in turn, is a function of (and increasing in) unemployment composition x_t (Panel D) and labor market tightness θ_t (Panel E). There is little difference in tightness, so the differences between IRFs are driven by unemployment composition.

After a negative productivity shock, firms post fewer jobs, which decreases tightness and the aggregate job finding rate. Due to pure duration dependence, those who do not find work may flow to the low state of unemployment. Since those in the low state find

[21] Results are computed using Dynare (Adjemian et al., 2011).

jobs at a lower rate, flows to the low state build over the course of the recession, resulting in the negative hump shape of x_t. Even though θ_t is recovering at the same speed, the composition effect drags down the recovery of the aggregate finding rate, and unemployment recovers more slowly. The effect is strong with more pure duration dependence. So, duration dependence is more pure, the unemployment rate is more persistent.[22]

Note that f_t mirrors the path of A_t in the DMP model. Herein lies the reason why the standard DMP model fails to generate persistence in unemployment. Since vacancies v_t is a jump variable, θ_t adjusts to A_t each period according to the free entry condition. So, θ_t follows the same path as A_t, and since the aggregate job finding rate is a function only of θ_t, the aggregate job finding rate follows the same path as A_t as well. The result is that the finding rate recovers as quickly as the shock.[23] My model avoids this result because f_t also a function of x_t. In fact, it is clear how f_t initially recovers quickly before getting "caught" by the slow-developing drop in x_t

Even without pure duration dependence, unemployment recovers more slowly with heterogeneity. In the pure heterogeneity model, after an initially positive effect on composition, high-state workers find jobs at a faster rate than low-state workers, worsening the pool and slightly slowing recovery.

Turning to stochastic simulations, Table 4 reports autocorrelations from simulating the model using a stochastic path for productivity A_t. In Table 4, the unemployment rate is significantly more persistent in the mixed model with an autocorrelation of 0.957 compared to 0.930 in the DMP model. The difference is a marked improvement given that the autocorrelation of unemployment is 0.975 in the data., though the persistence in the data is still significantly greater. And again, unemployment is more persistent with more pure duration dependence. In Table 4, the aggregate job finding rate is more persistent with pure duration dependence, and the greater persistence of the job finding rate increases the persistence of the unemployment rate. If the unemployment rate is

[22]For every line except for the case of pure duration dependence, x_t increases in the period after the shock. This has to do with the different levels values of γ, ϕ, and the steady state x across different calibrations.

[23]Table 8 in the Appendix shows that the correlation between A_t and f_t is 0.999 in the DMP model.

Table 4: Simulation autocorrelations from stochastic productivity

	Data	Model			
		DMP	Mixed	Pure duration dependence	Pure heterogeneity
$AC(u_t)$	0.975	0.930	0.957	0.965	0.944
$AC(\theta_t)$	0.969	0.878	0.892	0.904	0.883
$AC(f_t)$	0.980	0.878	0.929	0.948	0.901

First column is data, rest of columns are simulations of different models. Vacancy data source: JOLTS and Barnichon (2010). All other data: CPS. All series are quarterly and HP filtered with smoothing parameter 10^5.

modeled as an AR(1) process, then the half-life of the unemployment rate is 16 months in the mixed model compared to 9 months in the DMP model.[24]

I include more thorough IRFs and simulation results in Appendix E and Appendix D, respectively.

6.2 Separation Shocks

In contrast, duration dependence does not slow unemployment recovery after a recession if the recession is driven by a separation rate shock, as illustrated by the IRFs in Figure 8. Unlike a productivity shock, a positive separation shock induces a *positive* composition effect which offsets the negative effects of the shock.

As Panel A of Figure 8 shows, the separation rate shock triggers a wave of workers entering unemployment. Since $\zeta < 0.5$, most workers enter the high state of unemployment upon separation.[25] And since, by definition, newly-separated workers are short-term unemployed, the composition of the unemployment pool quickly jumps toward the high state in Panel D. The improved composition of the unemployment pool increases the probability that firms successfully hire workers. So, firms are incentivized to post more job vacancies, and θ_t spikes up. Since both x_t and θ_t increase, the aggregate job finding rate increases in Panel B, generating an increase in unemployment outflows which

[24]Using the autocorrelation coefficient ρ, the half-life is calculated using $\log(0.5)/\log(\rho)$ (Gorry et al., 2020). In the data, the half-life of unemployment is 27 months.

[25]$\zeta < 0.5$ for all models.

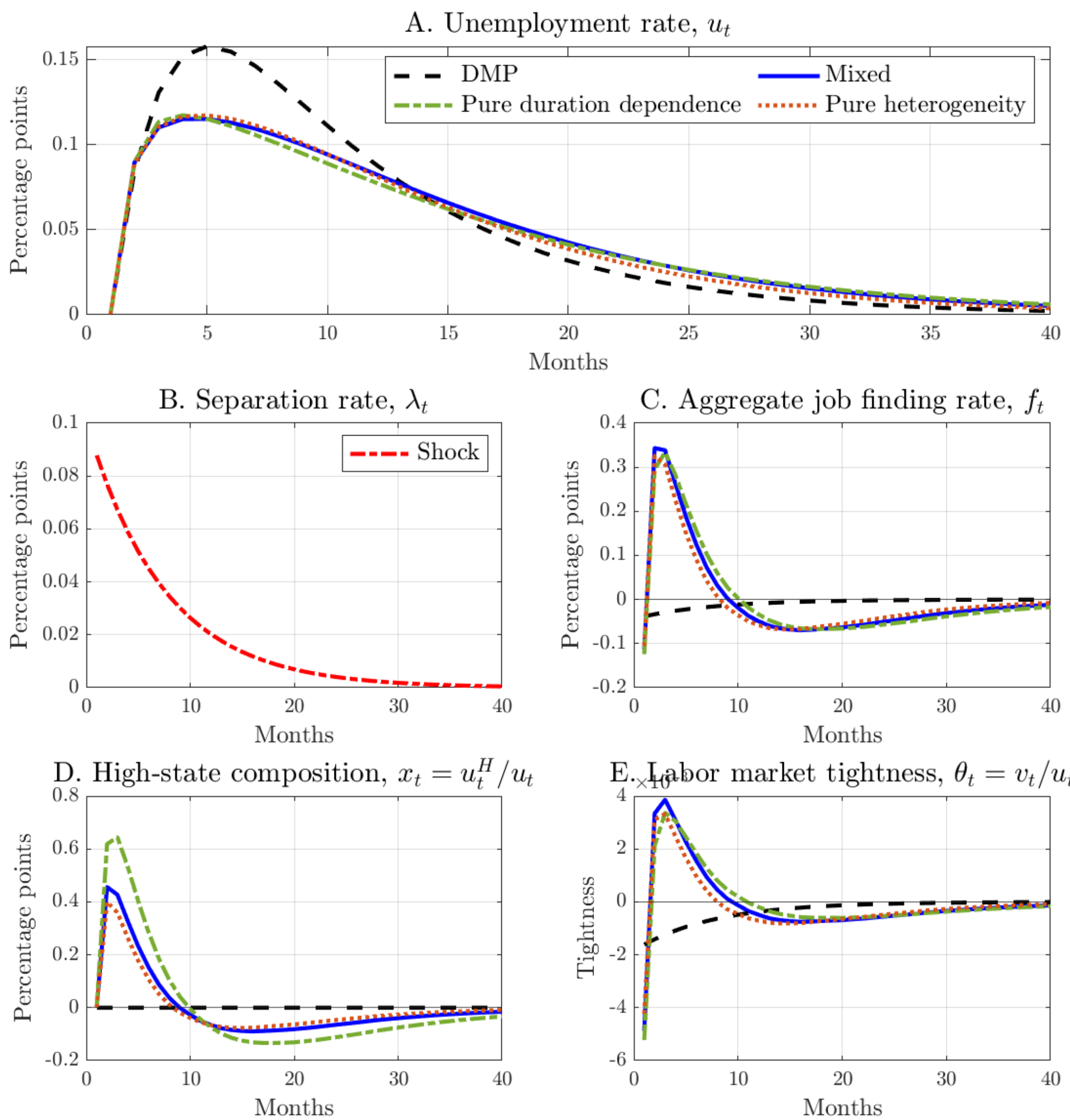

Figure 8: Impulse response functions after separation rate shock

Impulse response functions following a positive separation rate shock. All impulse responses are measured in deviations from the steady state. DMP refers to the the model without unemployment heterogeneity. The baseline calibration is a realistic combination of pure duration dependence and heterogeneity; the pure duration dependence calibration attributes all differences in job finding to duration dependence; and the pure heterogeneity calibration attributes all differences in job finding to heterogeneity. Panel B plots the exogenous shock; all other panels plot endogenous responses.

offsets the increase in inflows. In short, after a separation rate shock, a positive shift in composition spurs a job posting spree which mitigates the rise in unemployment.

In the long run, unemployment composition eventually decreases as unemployed workers shift to the low state. But the effect is small since the shock has mostly subsided.

7 Conclusion

The high persistence of the unemployment rate presents a challenge for the standard DMP search model. In a standard DMP model, there is little internal propagation, so the model does not generate persistence of the unemployment rate beyond the persistence of the shock. In this paper, I embed negative duration dependence into DMP model and show that accounting for negative duration dependence helps to reconcile the model with the data.

Unlike standard DMP models in which unemployed workers are homogeneous, my model includes two states of unemployment, high and low, where workers in the low state face lower job finding rates. A fraction of all workers are perpetually in the low state, but unemployed workers can also reach to the low state as result of pure duration dependence. Thus, the model can accommodate both explanation for why job finding rates decrease with unemployment duration: pure heterogeneity and duration dependence.

I calibrate the model so that it matches standard data moments as well as the shape of the job finding rate over unemployment duration. The model is flexible enough that it can match the same moments for infinite combinations of pure duration dependence and heterogeneity. I explore three model variants: a realistic mix of pure duration dependence and heterogeneity, and extreme cases with only pure duration dependence and only heterogeneity.

Intuitively, as job finding rates decrease during recessions, the composition of the unemployment pool shifts towards the long-term unemployed. As a result of pure duration dependence, the long-term unemployed are less likely to find jobs. Since the shifting of the unemployment pool is a relatively slow process, the aggregate job finding rate recovers more slowly. Crucially, the magnitude of the effect depends on the extent to which the negative relationship between job finding rates and unemployment duration is driven by pure duration dependence. Thus, this paper highlights the importance of distinguishing between pure duration dependence and heterogeneity for macroeconomists.

Appendix

A Data

To construct the moments used for calibration, I use CPS data for January 1978 - March 2020. Job finding rates are calculated by dividing unemployment to employment transitions over transitions from unemployment back to unemployment or to employment.

Comparing simulation results with the data also requires data for vacancies and productivity. For vacancies, I use JOLTS and, for before 2000, the composite help-wanted index from Barnichon (2010). For productivity, I use the BLS measure of labor productivity for all workers in the nonfarm business sector.

B Calibration Details

B.1 Targeting $f(\tau)$

I calibrate parameters to match the shape of the job finding rate as a function of unemployment duration, $f(\tau)$.

As is described in Appendix C, in the model's steady state, the probability of finding a job given unemployment duration τ

$$f(\tau) = \begin{cases} (1-\zeta)f^H + \zeta f^L, & \tau = 0 \\ \left(f^H - f^L\right)(1-\zeta)(1-\phi)^\tau \dfrac{\left(1-f^H\right)^\tau}{\prod_{i=0}^{\tau-1}\left(1-f(\tau-i)\right)} + f^L, & \tau \geq 1. \end{cases} \tag{16}$$

We can write Equation (16) as

$$f(\tau) = \begin{cases} a + c & \tau = 0 \\ ab^\tau \left(\dfrac{1}{1-f(0)}\right)\left(\dfrac{1}{1-f(1)}\right)\cdots\left(\dfrac{1}{1-f(\tau-1)}\right) + c, & \tau \geq 1. \end{cases} \tag{17}$$

where

$$a = \left(f^H - f^L\right)(1-\zeta), \tag{18}$$

$$b = (1-\phi)\left(1 - f^H\right), \tag{19}$$

and

$$c = f^L. \tag{20}$$

I first estimate the coefficients a, b, and c in the data. The result is the target curve in Figure 5. As Figure 5 shows, the functional form of Equation (17) fits the data well. I then target a, b, and c in calibration.[26]

Equations (18), (19) and (20) are intuitive. First, c is the horizontal asymptote of $f(\tau)$; as an worker's unemployment duration increases, their job finding probability converges downward to f^L. $a + c$ is the vertical intercept of $f(\tau)$, or the job finding probability of a worker within the month that they were separated from their previous job. So, $a + c = f(0) = (1-\zeta) f^H + \zeta f^L$. b determines the speed of convergence.

B.2 Targeting Estimates from van den Berg and van Ours (1996)

van den Berg and van Ours (1996) nonparametrically estimate the relative relative decrease in job finding rates from one month of unemployment to the next that is due to pure duration dependence. Borrowing notation, η_τ is the decrease in the job finding rate for the mass of unemployed workers from month $\tau - 1$ to τ that would occur if initial heterogeneity in job finding rates had no effect. In my model, removing heterogeneity is equivalent to setting ζ equal to zero, so this moment translates to

$$\eta_\tau = \frac{f(\tau \mid \zeta = 0)}{f(\tau - 1 \mid \zeta = 0)}. \tag{21}$$

η_τ is estimated for $\tau = 1, 2, 3$ in van den Berg and van Ours (1996), and I target all of them

[26]Since I am not targeting the data directly but rather an implication of the data, this method is can be described as indirect inference. Jarosch and Pilossoph (2019) and Kroft et al. (2016) use a similar strategy to fit $f(\tau)$.

Table 5: Steady state variables across models

Variable	Baseline	Pure duration dependence	Pure heterogeneity	Hagedorn-Manovskii
u	0.062	0.062	0.059	0.062
x	0.36	0.54	0.36	0.36
θ	0.54	0.54	0.53	1.11
w	0.99	0.99	0.99	0.97
f^H	0.55	0.43	0.58	0.54
f^L	0.18	0.19	0.19	0.18

Steady state variables across model calibrations.

Table 6: Calibration fit across model specifications

		Model			
Moment	Target	Mixed	Pure duration dependence	Pure heterogeneity	Hagedorn-Manovskii
u	0.062	0.062	0.062	0.059	0.062
f	0.31	0.32	0.32	0.33	0.32
$f(\tau)$					
a	0.25	0.25	0.25	0.24	0.25
b	0.41	0.40	0.40	0.42	0.40
c	0.20	0.18	0.19	0.19	0.18
η					
η_1	0.96	0.92			0.92
η_2	0.85	0.87			0.87
η_3	0.80	0.83			0.83

Simulated moments across model specifications.

in the baseline calibration.

B.3 Additional Calibration Results

Table 5 lists steady state variables and Table 6 describes the model fit across models. Both tables include the Hagedorn-Manovskii model. In the Hagedorn-Manovskii model, I follow Hagedorn and Manovskii (2008) and set $z = 0.955$ and $\eta = 0.052$. I then recalibrate the model, including the additional targets for the mixed model from van den Berg and van Ours (1996). The resulting parameters are listed in Table 7.

Table 7: Hagedorn-Manovskii calibrated parameters

Parameter	Meaning	DMP	Mixed	Hagedorn-Manovskii
γ	Low state penalty		0.34	0.13
ϕ	Transition rate	0.00	0.13	0.13
ζ	Initial low state	0.00	0.31	0.31
μ	Match efficiency	0.40	0.98	0.73

Compares Hagedorn-Manovski calibrated parameters with calibrations from Table 2.

C Deriving $f(\tau)$

Let $u_t(\tau)$ denote the number of unemployed workers who have been unemployed for τ continuous periods. Note that $\sum_\tau u_t(\tau) = u_t$. Let $u_t^H(\tau)$ denote the number of unemployed workers who have been unemployed for τ periods and are in the high state. Finally, let $x_t(\tau)$ denote the fraction of high-state unemployed workers among all unemployed workers with τ periods of unemployment, $x_t(\tau) = u_t^H(\tau)/u_t(\tau)$. Terms without subscripts refer to the steady state.

To derive $f(\tau)$, I must first derive $x(\tau)$. The number of unemployed workers who have not yet been unemployed for a full period ($\tau = 0$) equals the number of workers who were just separated from their jobs. So,

$$u_t(0) = \lambda_t(1 - u_t), \tag{22}$$

The number of unemployed workers in the high state with $\tau = 0$ is

$$u_t^H(0) = (1 - \zeta)\lambda_t(1 - u_t). \tag{23}$$

Equations (22) and (23) imply

$$x_t(0) = 1 - \zeta.$$

The number of unemployed workers who have been unemployed for one period ($\tau = 1$) consists of the worker who were separated in the previous period and did not find a job within that period. Therefore,

$$u_t(1) = u_{t-1}(0)(1 - f_{t-1}(0)).$$

Plugging in Equation (22), we have

$$u_t(1) = \lambda_{t-1}(1 - u_{t-1})(1 - f_{t-1}(0)). \tag{24}$$

Similarly, the number of workers in the high state with $\tau = 1$ is

$$u_t^H(1) = u_{t-1}^H(0)\left(1 - f_{t-1}^H(0)\right)(1 - \phi).$$

Note that we include $(1 - \phi)$ to account for workers who flow to the low state. Plugging in Equation (23), we have

$$u_t^H(1) = (1 - \zeta)\lambda_{t-1}(1 - u_{t-1})\left(1 - f_{t-1}^H(0)\right)(1 - \phi).$$

Since the job finding rate in the high state is not a function of unemployment duration, $f_t^H(\tau) = f_t^H$ for all τ. So, $u_t^H(1)$ can be written as

$$u_t^H(1) = (1 - \zeta)\lambda_{t-1}(1 - u_{t-1})\left(1 - f_{t-1}^H\right)(1 - \phi). \tag{25}$$

Equations (24) and (25) imply

$$x_t(1) = (1 - \zeta)(1 - \phi)\frac{1 - f_{t-1}^H}{1 - f_{t-1}(0)}.$$

Using the same method for $\tau = 2$, we have

$$x_t(2) = (1 - \zeta)(1 - \phi)^2\left(\frac{1 - f_{t-2}^H}{1 - f_{t-2}(0)}\right)\left(\frac{1 - f_{t-1}^H}{1 - f_{t-1}(1)}\right)$$

The pattern continues for all $\tau \geq 3$.

In summary, for $\tau \geq 1$,

$$x_t(\tau) = (1-\zeta)(1-\phi)^{\tau} \left(\frac{1-f_{t-\tau}^{H}}{1-f_{t-\tau}(0)} \right) \left(\frac{1-f_{t-\tau+1}^{H}}{1-f_{t-\tau+1}(1)} \right) \cdots \left(\frac{1-f_{t-1}^{H}}{1-f_{t-1}(\tau-1)} \right).$$

Concisely written, we have

$$x_t(\tau) = \begin{cases} 1-\zeta, & \tau = 0 \\ (1-\zeta)(1-\phi)^{\tau} \prod_{i=0}^{\tau-1} \frac{1-f_{t-i}^{H}}{1-f_{t-i}(\tau-i)}, & \tau \geq 1. \end{cases} \tag{26}$$

In the steady state,

$$x(\tau) = \begin{cases} 1-\zeta, & \tau = 0 \\ (1-\zeta)(1-\phi)^{\tau} \frac{\left(1-f^{H}\right)^{\tau}}{\prod_{i=0}^{\tau-1}\left(1-f(\tau-i)\right)}, & \tau \geq 1. \end{cases}$$

Note the asymptotic properties of $x(\tau)$. Since $f^H > f(\tau)$ for all τ, $x(\tau) \to 0$ as $\tau \to \infty$. In words, at longer unemployment lengths, the unemployment pool becomes dominated by the low-state workers.

The job finding probability of a worker who has been unemployed for τ periods is the weighted average for f_t^H and f_t^L weighted by the probability that the worker is in the high or low state:

$$f_t(\tau) = f_t^H x_t(\tau) + f_t^L(1-x_t(\tau)) = x_t(\tau)\left(f_t^H - f_t^L\right) + f_t^L.$$

Since $x(\tau) \to 0$ as $\tau \to \infty$, $f(\tau) \to f^L$ as $\tau \to \infty$; as a worker's unemployment duration increases, his job finding rate converges to f_t^L.

Plugging in $x_t(\tau)$ from Equation (26), we have

$$f_t(\tau) = \begin{cases} (1-\zeta) f_t^H + \zeta f_t^L, & \tau = 0 \\ \left(f_t^H - f_t^L\right)(1-\zeta)(1-\phi)^\tau \prod_{i=0}^{\tau-1} \frac{1 - f_{t-i}^H}{1 - f_{t-i}(\tau - i)} + f_t^L, & \tau \geq 1. \end{cases}$$

In the steady state,

$$f(\tau) = \begin{cases} (1-\zeta) f^H + \zeta f^L, & \tau = 0 \\ \left(f^H - f^L\right)(1-\zeta)(1-\phi)^\tau \frac{(1-f^H)^\tau}{\prod_{i=0}^{\tau-1}(1 - f(\tau - i))} + f^L, & \tau \geq 1. \end{cases} \tag{27}$$

D Simulation Tables

Table 8 reports simulation statistics comparing the data and simulated models with an exogenous productivity process. For each, I list the standard deviation, autocorrelation, and correlations between variables.[27] The autocorrelation numbers match those in Table 4.

Neither of the simulations in Table 8 come close to generating the volatility of unemployment in the data (Shimer, 2005). As suggested by Hagedorn and Manovskii (2008), one way to resolve the puzzle is to use significantly different parameter values for the opportunity cost of employment z and the wage bargaining parameter ψ. I follow this logic in the Hagedorn-Manovskii model. Indeed, using the new parameter values increases the volatility of unemployment; the persistence results are unaffected.

E Additional Impulse Response Functions

In this section, I add wages, output, and vacancies to the IRFs in Section 6. Table 9 illustrates the effects of productivity shocks and Table 10 illustrates the effects of separation

[27] The correlation between productivity and the unemployment rate is significantly weaker in the data than the model. This has to do with my time period, 1978-2019, and is pointed out in Hagedorn and Manovskii (2011).

Table 8: Simulation statistics from stochastic productivity

		u_t	v_t	θ_t	f_t	A_t
Data						
St. dev.		0.198	0.200	0.385	0.143	0.017
Autocorr.		0.975	0.959	0.969	0.980	0.927
	u_t	1	-0.873	-0.944	-0.988	0.242
	v_t		1	0.979	0.888	-0.207
Correlation	θ_t			1	0.946	-0.214
	f_t				1	-0.275
	A_t					1
DMP model simulation						
St. dev.		0.025	0.049	0.072	0.029	0.020
Autocorr.		0.930	0.798	0.878	0.878	0.877
	u_t	1	-0.831	-0.924	-0.924	-0.923
	v_t		1	0.980	0.981	0.979
Correlation	θ_t			1	1.000	0.998
	f_t				1	0.999
	A_t					1
Mixed model simulation						
St. dev.		0.028	0.045	0.068	0.031	0.020
Autocorr.		0.957	0.802	0.892	0.929	0.877
	u_t	1	-0.721	-0.889	-0.952	-0.857
	v_t		1	0.958	0.895	0.972
Correlation	θ_t			1	0.984	0.996
	f_t				1	0.968
	A_t					1
Hagedorn-Manovskii model simulation						
St. dev.		0.151	0.328	0.760	0.208	0.020
Autocorr.		0.961	0.783	0.801	0.943	0.877
	u_t	1	-0.610	-0.464	-0.829	-0.816
	v_t		1	0.415	0.886	0.878
Correlation	θ_t			1	0.469	0.545
	f_t				1	0.898
	A_t					1

Top panel is data and the rest of the panels are from simulations of model variants.. The top two rows of each panel list the standard deviation and autocorrelation of each variable. The rest of the panel is the contemporaneous correlation between two variables. All series are quarterly and HP filtered with smoothing parameter 10^5.

rate shocks.

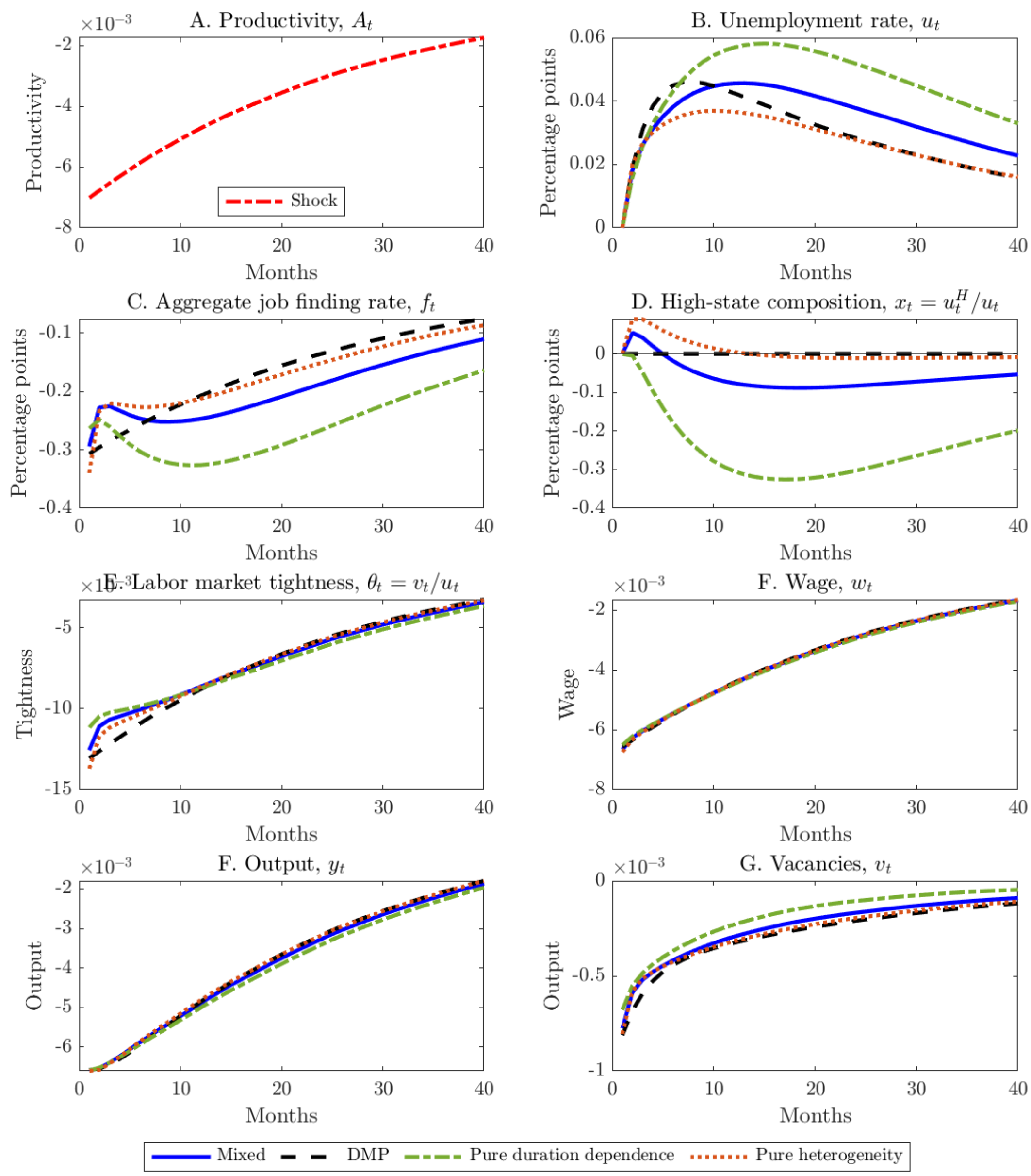

Figure 9: Complete impulse response functions after negative productivity shock

Impulse response functions following a negative productivity shock. All impulse responses are measured in deviations from the steady state. Panel A plots the exogenous shock; the other panels plot endogenous responses. These IRFs build upon the IRFs in Figures 7. Output is defined as $y_t = A_t n_t$.

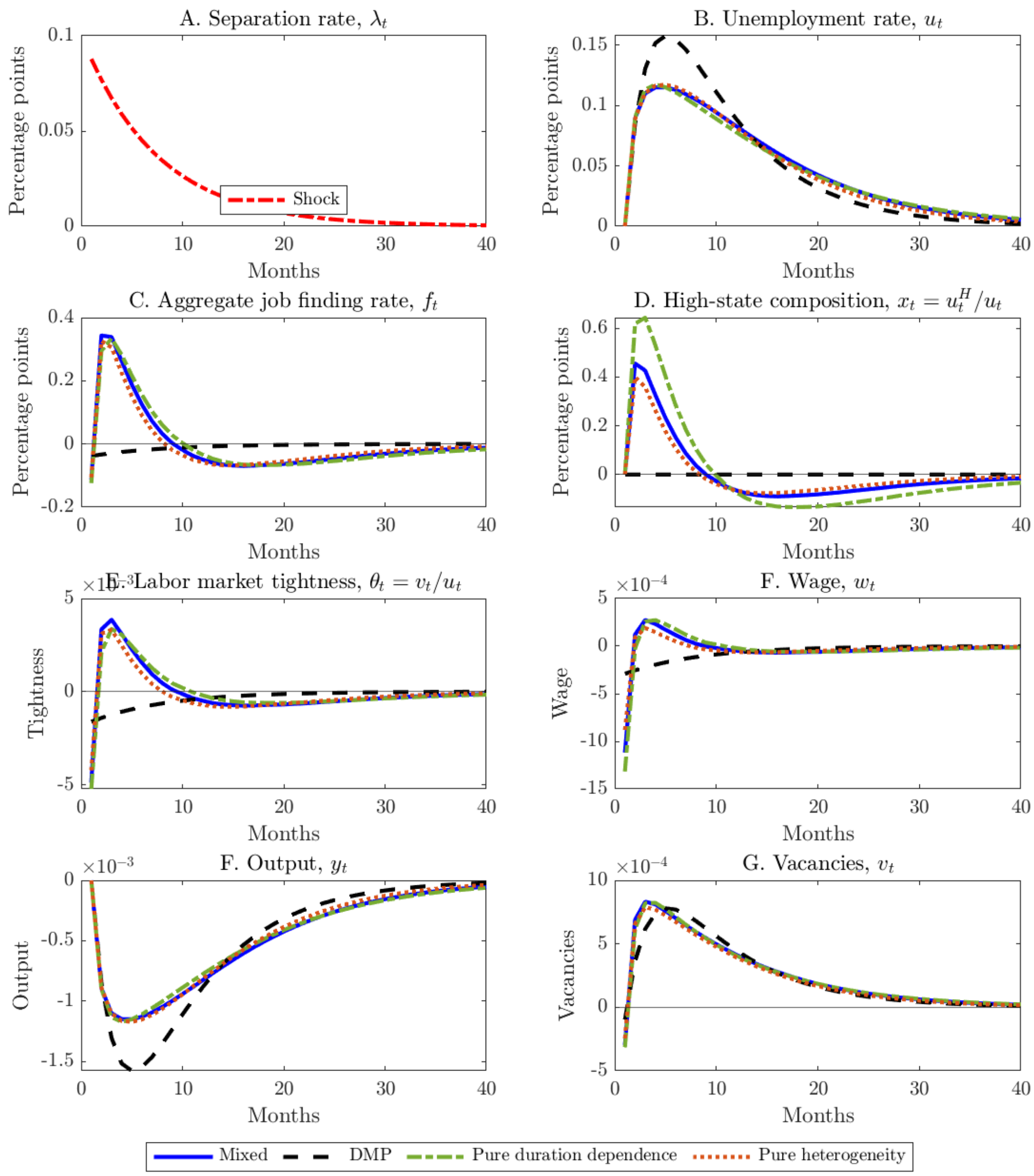

Figure 10: Complete impulse response functions after positive separation rate shock

Impulse response functions following a positive separation rate shock. All impulse responses are measured in deviations from the steady state. Panel A plots the exogenous shock; the other panels plot endogenous responses. These IRFs build upon the IRFs in Figure 8. Output is defined as $y_t = A_t n_t$.

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